The design of a simulated in-line side-coupled 6 MV linear accelerator waveguide

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Purpose: The design of a 3D in-line side-coupled 6 MV linac waveguide for medical use is given, and
the effect of the side-coupling and port irises on the radio frequency (RF), beam dynamics, and
dosimetric solutions is examined. This work was motivated by our research on a linac-MR hybrid
system, where accurate electron trajectory information for a clinical medical waveguide in the
presence of an external magnetic field was needed.

Methods: For this work, the design of the linac waveguide was generated using the finite element
method. The design outlined here incorporates the necessary geometric changes needed to incor-
porate a full-end accelerating cavity with a single-coupling iris, a waveguide-cavity coupling port
iris that allows power transfer into the waveguide from the magnetron, as well as a method to
control the RF field magnitude within the first half accelerating cavity into which the electrons from
the gun are injected.

Results: With the full waveguide designed to resonate at 2998.5 ± 0.1 MHz, a full 3D RF field
solution was obtained. The accuracy of the 3D RF field solution was estimated through a compari-
son of important linac parameters (Q factor, shunt impedance, transit time factor, and resonant
frequency) calculated for one accelerating cavity with the benchmarked program SUPERFISH. It was
found that the maximum difference between the 3D solution and SUPERFISH was less than 0.03%.
The eigenvalue solver, which determines the resonant frequencies of the 3D side-coupled wave-
guide simulation, was shown to be highly accurate through a comparison with lumped circuit
theory. Two different waveguide geometries were examined, one incorporating a 0.5 mm first side
cavity shift and another with a 1.5 mm first side cavity shift. The asymmetrically placed side-
coupling irises and the port iris for both models were shown to introduce asymmetries in the RF
field large enough to cause a peak shift and skewing of an
initially cylindrically uniform electron beam accelerating within the waveguide. The shifting and
skewing of the electron beam were found to be greatest due to the effects of the side-coupling irises
on the RF field. A further Monte Carlo study showed that this effect translated into a 1% asymmetry
in a 40 × 40 cm² field dose profile.

Conclusions: A full 3D design for an in-line side-coupled 6 MV linear accelerator that emulates a
common commercial waveguide has been given. The effect of the side coupling on the dose
distribution has been shown to create a slight asymmetry, but overall does not affect the clinical
applicability of the linac. The 3D in-line side-coupled linac model further provides a tool for the
investigation of linac performance within an external magnetic field, which exists in an integrated
linac-MR system. © 2010 American Association of Physicists in Medicine.

Key words: linear accelerator, finite element method, particle simulation, waveguide design

I. INTRODUCTION

Current practice in performing image guided radiotherapy (IGRT) is to image a patient immediately prior to radiation
treatment in order to identify patient setup variations as well as tumor and organs at risk locations. The difficulty with the
current IGRT practice is that organ, tumor, and patient mo-
ation during treatment cannot be accounted for due to the limitations of the imaging modalities currently available; no imaging system exists that can image the patient in three-dimensions (3D) during treatment. To properly address intratreatment motion, our group is developing a linac-MR system using a standard in-line side-coupled 6 MV linac waveguide coupled to an 70 cm bore biplanar magnetic resonance (MR) imager. A working small scale linac-MR prototype has already been built by our group using a 6 MV x-ray source and a 27.9 cm magnet bore, where this linac-MR design provides a method to image the tumor site and organs at risk in real time during irradiation in 3D. This new technology will make 3D tumor tracking and dose adaptation during treatment possible in real time, allowing for dose escalation at the tumor site while decreasing unwanted dose to normal tissues; both of which are expected to lead to improved survival probabilities. The proposed linac-MR system allows the x-rays produced by the linac to pass between the magnet plates of the biplanar magnet. With this configuration, magnetic fringe fields from the biplanar magnets are far-reaching and will intersect the entire linac waveguide (including the electron gun) in a direction perpendicular to its length. With no magnetic shielding, these fringe fields will deflect the electrons in the linac from their original straight path causing large x-ray field asymmetries, substantial beam loss, and a clinically nonusable radiation beam. In order to accurately quantify the extent of these deflections and to later determine the requirements for magnetic shielding, a full 3D simulation of an in-line side-coupled 6 MV medical linac is required.

The modeling and optimization of linear accelerator cavities have been performed numerically for many decades using various software; however, to date, no complete 3D electron linac simulation for medical purposes has been published. A more common and popular choice of program is the axisymmetric two-dimensional (2D) finite difference (FD) program SUPERFISH (Los Alamos National Laboratory, NM). SUPERFISH has been used in the design of axisymmetric waveguide systems and through benchmarking has been shown to be highly accurate. This 2D software, however, is unable to fully model side-coupled waveguide systems due to its requirement for axisymmetric cavity designs. With the increasing power of personal computers, full 3D simulations have become available with a variety of numerical methods to solve Maxwell’s equations which are capable of solving side-coupled systems. In many cases, however, the design and optimization of a full waveguide from a 3D numerical simulation are unnecessary and impractical due to extremely large computational requirements of memory and time for long structures with many beamline components. In these cases, only a section of the waveguide is investigated through simulation, and the results are typically compared to measurements using a prototype waveguide built from aluminum or “cold model.” When modeling only sections of the linac waveguide at a time, the electron beam characteristics can only be determined within each section of the waveguide modeled separately. Since in this work the electron dynamics within the entire waveguide is needed, a full 3D model incorporating all aspects of waveguide design and coupling is required.

In order to accurately determine the electron trajectories within an in-line side-coupled 6 MV medical linac waveguide, a full 3D waveguide model was designed to ensure that the simulated structure resonated at the appropriate frequency. The design of the 3D waveguide began by creating an accelerating cavity that emulates the characteristics of a commercial medical waveguide. The theory for coupled cavity waveguide design as linear accelerators and its coupling effects has been applied for a proton linac previously, and now is applied practically for the electron medical linac modeled here. This work provides a concise 3D simulation design for an in-line side-coupled 6 MV linac waveguide, and produces a radio frequency (RF) field solution incorporating the effects of side coupling.

II. METHODS

II.A. Waveguide design and RF simulations

II.A.1. Theoretical foundation for waveguide-cavity design

The theoretical foundation for designing each cavity to resonate at a nominal frequency was taken from the lumped circuit model of a side-coupled linear accelerator. A side-coupled linac is magnetically coupled with its coupling irises at a location of high magnetic field and low electric field, where power is transferred from one cavity to the next through mutual inductance. According to Slater’s perturbation formula, the introduction of a coupling iris in the location of a strong magnetic field causes a decrease in the resonant frequency of the cavity, and more generally, any geometric change in the cavity will cause a shift in its resonant frequency. As a simple approximation, referring to the lumped circuit model, each cavity’s resonant frequency squared is inversely proportional to the inductance of the cavity L and the cavity’s capacitance C,

\[ \omega^2 \propto \frac{1}{LC}. \]  

(1)

Slater’s perturbation theory can then be simply understood in this case, for example, by considering that the introduction of a coupling iris increases the inductance, which, in turn, decreases the resonant frequency. In general, in order to recover a resonant frequency of 2998.5 ± 0.1 MHz after the introduction, removal, or alteration of a coupling iris, the capacitance of the accelerating cavities was adjusted by changing the length of the nose cones, and the inductance was changed by altering the cavity diameter. In the side-coupling cavities, the only change required was to the capacitance of the cavity which was achieved by adjusting the post lengths.

II.A.2. Two-dimensional finite difference simulations

This work aimed at emulating a Varian 600C linac waveguide. The dimensions of a single accelerating cavity published by Roy and Shanker in 1993 (Ref. 25) was redesigned.
using the 2D FD program SUPERFISH, which offered very fast simulations but was restricted to axisymmetric cavities. The SUPERFISH mesh consisted of 7428 nodes of which 474 resided on the boundary where the nodes specify the triangles of the domain discretization with an average area of $5.25 \times 10^{-4}$ cm$^2$. Using the exact dimensions from Roy and Shanker, it was found that the beam hole diameter, as well as the effective shunt impedance (a measure of how well a waveguide works as a linear accelerator), was not identical to the Varian 600C waveguide. The accelerating cavity was thus redesigned by our group in order to achieve a beam hole diameter and approximate effective shunt impedance of 5 mm and 115 MΩ/m, respectively, which is consistent with a Varian 600C waveguide that achieves a nominal electron energy of 6 MeV. Since the effective shunt impedance of a simulated accelerating cavity is approximately 15% greater than a manufactured waveguide, the simulated accelerating cavity was designed to have an effective shunt impedance of 115 MΩ/m instead of the published 100 MΩ/m. Using this slightly larger value, the cavity geometry would give cavity dimensions that more closely approximated the manufactured waveguide.

**II.A.3. Three-dimensional FEM waveguide simulation**

The modified and optimized 2D design of the accelerating cavity was then reproduced in 3D using the finite element method (FEM) program COMSOL MULTIPHYSICS (Burlington, MA). The 3D mesh was generated using isoparametric tetrahedral elements with cubic vector shape functions to obtain smooth field solutions with increased accuracy and a port power of 2.3 MW was used. The resonant frequency of the side-coupled in-line linac waveguide was chosen to be 2998.5 MHz, which is within the frequency range of a standard e2V tunable 5-band magnetron and will hereafter be referred to as the nominal resonant frequency. The reference 3D accelerating cavity solution compared to the 2D SUPERFISH solution contained 55 501 mesh elements while the full in-line side-coupled linac waveguide model used 368 625 elements. With the reference cavity having 55 501 mesh elements, it had a very similar mesh to the full model, meaning the accuracy of the solution within the reference cell could be extrapolated to the solution of the full waveguide model. Using cubic vector elements, these mesh values translated into 255 305 nodes for the reference cavity mesh and just under 1.6 million nodes for the full waveguide model. The FEM RF field solution for the full waveguide model required 115 Gbytes of RAM and took just over an hour on a PC possessing 128 Gbytes of RAM running on 4 2.0 GHz quad-core AMD Opteron 8350 processors.

The full 3D linac waveguide simulation was completed in five design stages. Before each stage was complete, all the cavities were required to resonate at the required resonant frequency. The first design stage was of the “basic unit” shown in Fig. 1. With an overlap between the side-coupling and accelerating cavities of 6 mm (in accordance to a 2998 MHz frequency of Roy and Shanker), the SUPERFISH designed cavity diameter was adjusted slightly to account for the frequency reduction caused by the coupling irises. The numbers given in Fig. 1 represent dimensions published by Roy and Shanker, which were kept identical in this model, while the Greek letters correspond to dimensions that required optimization. The geometry of the basic unit, although not respecting the staggered up-down sequence seen in Fig. 2, is sufficient in order to design each cavity to resonate at the nominal frequency. The second design stage had the basic unit repeated five times, staggering the coupling cavities above and below the beam tube axis, as seen in Fig. 2. This created a waveguide terminated in half accelerating cavities at both ends. Since the basic unit was initially designed to resonate at 2998.5 ± 0.1 MHz for the TM$_{010}$ mode, no additional redesign of the waveguide shown in Fig. 2 was required. The third design stage required designing a full-end cavity with only one coupling iris. This was performed to...
ensure that the simulated 3D linac waveguide emulated a manufactured one with a full-end cavity. The fourth design stage required shifting the first side cavity (SC1 in Fig. 2) toward the front end of the waveguide where the electron gun is located. By shifting SC1 toward the front end of the waveguide, iris 1 becomes larger and iris 2 becomes smaller. This asymmetry in the coupling irises causes the RF field magnitude in the first accelerating cavity (AC1) to become smaller than that in the second accelerating cavity (AC2). The larger the asymmetry in the coupling irises, the smaller the RF field in AC1 becomes compared to AC2 (for the same input power). Thus, through the design of the first side cavity shift, control of the RF field magnitude in AC1 can be achieved. This is important since the RF field magnitude in AC1 plays a critical role in the extent of the injected electron beam blooming. Two different side cavity shifts of 0.5 and 1.5 mm will be examined in this paper. The fifth and last design stage is the design of a coupling port that feeds power from the magnetron, through the transmission waveguide into the linac waveguide. In 1995, Zhao et al. published a formula that determines the optimal coupling coefficient from a known input power with the goal of maximizing dose rate. With an input power of 2.3 MW, the optimal coupling coefficient was calculated to be 2.11. Since the coupling coefficient between the transmission waveguide and the linac waveguide is identical to the voltage standing wave ratio (VSWR) in the overcoupled regime (coupling coefficient greater than 1), the VSWR was calculated in the simulations. The VSWR measured is, in fact, a sum of two traveling waves, the power emission from the field within the linac waveguide at resonance and the reflected power at the port.\footnote{St. Aubin, Steciw, and Fallone: Design of a simulated 6 MV linac waveguide}

II.A.4. Determining the effects of the coupling ports on the RF field solution

The effect of the asymmetrically placed coupling and port irises in a side-coupled waveguide on the RF field was investigated next. In order to remove any numerical noise resulting from different FEM meshes in the comparison between coupled and noncoupled cavities, the RF field solution was solved on the same mesh for both the coupled and noncoupled cavities. Once the initial mesh was generated, the coupled solution was found using an eigenvalue solver with the boundary conditions at the irises set to require continuity in the field solution across the boundary. With the continuity condition enforced, the power is transferred from an accelerating cavity to a side cavity or from the magnetron into the linac waveguide just as in true operation. When the noncoupled cavity was solved, the iris boundary condition was simply changed from a continuity condition to a perfect electrical conductor which fully contained the RF field within the cavity. A difference map was then made with the coupled solution subtracted from the noncoupled solution such that any increases in field due to coupling resulted in positive values in the difference map. The coupled and noncoupled solutions were initially normalized to the average electric field on the axis to remove any differences in field magnitudes due to the eigenvalue solver. The normalized difference field was multiplied by a factor of 26 MV/m to obtain a representative field magnitude that would accelerate the electrons to 6 MeV and the difference maps were generated in each cavity at the same phase of the RF field for consistency in the analysis.

In order to determine that the difference maps generated as explained above were strictly due to the introduction of the irises and not due to the inherent differences in the field distributions caused by the RF at different frequencies (comparing the cavity frequency with and without irises), COMSOL was used to calculate changes in the field distributions due to the changes in frequency. The same cavity geometry and mesh was used in the analysis with the RF frequency simply changed to the cavity frequency determined with and without irises.

II.B. Particle simulations

The electron phase space at the target of the in-line 6 MV linac waveguide model was calculated using the “phase and radial motion in electron linear accelerators” (PARMELA)\footnote{St. Aubin, Steciw, and Fallone: Design of a simulated 6 MV linac waveguide} 3D particle tracking program (Los Alamos National Laboratory, NM) which calculates electron acceleration/trajectories in linear accelerators due to an RF field. The PARMELA simulation includes the effects of space charge (fields generated by the electron beam itself) as well as a calculation of beam loading. To quantify the effect of the coupling and port irises on the electron beam, PARMELA was used to calculate the electron trajectories within the two waveguide geometries investigated (0.5 and 1.5 mm side cavity shift) as well as for a fictitious waveguide with no coupling. In order to simulate a waveguide with no coupling, the 3D field within a noncoupled axisymmetric accelerating cavity was solved using the FEM and repeated many times to create the same number of cavities as the coupled system. A 15 keV cylindrical beam comprised of four million macroparticles, each representing 108 electrons, was tracked through the linac for all PARMELA simulations over two RF periods. The maximum energy of the electrons impinging on the target was set to 6 MeV and only the central electron bunch was used in the analysis disregarding the first and last half electron bunches to remove all simulation end effects so that the analyzed beam represented a steady-state electron bunch. In order to generate a sufficiently large phase space for the Monte Carlo studies, each PARMELA simulation was run 60 times (yielding roughly 40 million particles in total) with randomized electron injection locations within the cylindrical volume to ensure uncorrelated results.

II.C. Monte Carlo simulations

A Varian 600C linac head was modeled in the BEAMnrcMP 2007 (BEAM) Monte Carlo code according to the information supplied from the manufacturer. Directional bremsstrahlung splitting was used with a source to surface distance of 100 cm, a splitting radius of 40 cm, and a splitting factor of 1000 for a 40 × 40 cm² field size. The values of electron (ECUT) and photon (PCUT) transport cutoff energies were 0.70 and 0.01 MeV, respectively, and range rejection was
TABLE I. Computed values for some important linac parameters for both the 2D FD program SUPERFISH and the 3D FEM program COMSOL MULTIPHYSICS. Note that there was no change in the transit time factor in either the SUPERFISH or COMSOL simulation to the fourth decimal upon the maximum reduction in mesh size.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SUPERFISH</th>
<th>COMSOL</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ factor (rad)</td>
<td>17.523 ± 4</td>
<td>17.521 ± 0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Shunt impedance ($\Omega$/m)</td>
<td>165.3 ± 0.1</td>
<td>165.24 ± 0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Transit time factor</td>
<td>0.8379</td>
<td>0.8381</td>
<td>0.03</td>
</tr>
<tr>
<td>Resonant frequency (MHz)</td>
<td>3007.6 ± 0.2</td>
<td>3007.23 ± 0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

III. RESULTS AND DISCUSSION

III.A. 3D accelerating cavity and RF field solution benchmarking

Since our group does not have the resources to produce a cold model for verification of the RF field solution, validation was performed with a comparison against the benchmarked program SUPERFISH for a single accelerating cavity, and with theory from the lumped circuit model as seen later in this paper. The results from the redesigned and optimized 2D accelerating cavity reproduced in 3D using COMSOL MULTIPHYSICS is seen in Table I, where the important linac parameters ($Q$ factor, shunt impedance, transit time factor, and resonant frequency) are compared to those generated using SUPERFISH. The results show a maximum discrepancy of 0.03% between the two methods of calculation, showing excellent agreement. The tolerances given in Table I for the SUPERFISH simulation were determined by reducing the average triangle area in steps down to a final value of $5.04 \times 10^{-5}$ cm$^2$ (thus, increasing the number of nodes to a maximum of 75,636) and calculating the difference in the parameters at each step. The same technique was used for the COMSOL simulations where the mesh size was decreased by roughly five times giving 338,360 elements (and 1.55 million nodes). The lower tolerances in the COMSOL solution can be understood due to COMSOL using isoparametric elements to better conform to the boundary and cubic shape functions for higher order interpolation.

III.B. Waveguide cavity design

SUPERFISH was used in the design and optimization of all aspects of the accelerating cavity except for the cavity diameter which was optimized using the 3D FEM to account for the frequency change due to the coupling irises. It was found that the resonant frequency of the uncoupled accelerating cavity needed to be 3007.2 MHz such that when coupled to side cavities with an overlap of $d=6$ mm (see Fig. 1), the system of cavities, and hence the entire linac waveguide, would resonate at 2998.5 MHz. With the overlap between the side and accelerating cavities set to 6 mm, a coupling coefficient of 0.0112 was calculated, in good agreement with previous simulations and measurement taken by Roy and Shanker.

With each accelerating and side cavity individually resonating at 2998.5 ± 0.1 MHz, the waveguide system terminating in half cavities shown in Fig. 2 resonated at 11 different frequencies corresponding to 11 different phase shifts per cavity for the TM$_{010}$ mode. The 11 frequencies plotted against the 11 phase shifts per cavity produce a dispersion curve that was compared to the theoretical curve from lumped circuit theory in Fig. 3(a) using the calculated coupling coefficient given above. The resonant frequencies, calculated using COMSOL’s eigenfrequency solver, show excellent agreement with theory. From the dispersion curve, the phase velocity was determined to be speed of light ($c$) and the group velocity was determined to be 0.03$c$, in agreement with the values for a Varian 600C waveguide. The axial field calculated within the waveguide in Fig. 2 is given in Fig. 3(b).

The design of the full-end cavity and the cavities affected by the first side cavity shift were accomplished next in order to create a waveguide geometry that better represented a manufactured waveguide. For the end full cavity design, it was found that an increase of 0.145 mm increased the capacitance of that cavity enough to compensate for the single coupling iris, and thus regain the nominal resonant frequency. A decrease in the RF field magnitude within the first half accelerating cavity was controlled by the magnitude of the first side cavity shift toward the front of the waveguide. Figure 4 shows the axial ($z$ direction) electric field within the first half and second full cavity with a 0.5 mm side cavity.
shift and with a 1.5 mm side cavity shift. A clear drop in the field magnitude is seen in the first half accelerating cavity for the 1.5 mm cavity shift compared to the 0.5 mm shift. The total energies contained within the first half accelerating cavity were calculated to be 0.163 and 0.052 J for the 0.5 and 1.5 mm side cavity shifts, respectively. However, the total energy contained within the first coupling cavity remained constant at around 0.001 J for either cavity shift (roughly three orders of magnitude smaller than the accelerating cavity). The larger coupling iris of the first half accelerating cavity caused an increased inductance, meaning a decreased capacitance was required to regain the nominal resonant frequency. For the 1.5 mm cavity shift, the decreased capacitance was obtained by shortening the first nose cone by 0.03 mm, while for the 0.5 mm cavity shift, no nose cone adjustment was required since the frequency change was less than 0.1 MHz. The side cavity also experienced an overall increase in inductance, so in order to decrease capacitance, the lengths of the posts were reduced by 0.03 mm for the 1.5 mm cavity shift geometry, while again no change was required for the 0.5 mm cavity shift.

The input port cavity through which the power from the magnetron is fed required a redesigning of the accelerating cavity due to the introduction of the optimized port iris which gave a coupling coefficient of 2.09. The size of the input coupling port was different for the different side cavity shift geometries since the shifts changed the intrinsic impedance of the waveguide due to the different geometries. For the 1.5 mm cavity shift geometry with a coupling port area of 3.05 cm², the accelerating cavity diameter was decreased by 0.135 mm to recover the nominal resonant frequency since the addition of the port iris initially caused a 9.6 MHz decrease in frequency. Decreasing the diameter of this cavity had the effect of changing the intersection between it and the adjacent side cavities causing smaller coupling irises, but the effective change in the iris size was found to be so minor that no significant change (>0.1 MHz) in the resonant frequency of the side cavities was observed. For the 0.5 mm side cavity shift geometry with a coupling port area of 3.73 cm², the accelerating cavity diameter was decreased by 0.195 mm to compensate for the 13.8 MHz decrease in frequency and the side cavity’s posts were decreased by 0.01 mm to recover the side cavity’s nominal resonance frequency. The total energy within the port accelerating cavity was calculated to be 0.33 J, while in the adjacent coupling cavities, it was calculated to be 0.001 J for both the 1.5 and 0.5 mm side cavity shift designs. With all design optimizations completed, the RF field within the fully modeled waveguide was solved. Figure 5 shows the full 3D waveguide design together with the coupling port geometry, while Fig. 6 shows the axial (z direction) electric field FEM solution on axis along with the radial electric field at the beam tube edge for both side cavity shift in-line 6 MV linac waveguides. A summary of the waveguide dimensions along with optimization and design results is given in Table II.

III.C. Effect of side coupling on RF field and electron beam

The asymmetry in the RF field caused by the side and port coupling irises was investigated in the third and forth accelerating cavities shown as AC3 and AC4 in Fig. 5. The third accelerating cavity (AC3) possesses two side-coupling irises and the port cavity (AC4) possesses two side-coupling irises along with the port iris. The frequency difference between the accelerating cavity (AC3) with and without coupling was calculated to be 8.75 MHz, while the frequency difference for the port cavity was calculated to be 13.84 MHz (due to the reduced diameter required maintain the nominal resonance frequency with the port iris). It was determined through the COMSOL simulations that the magnitude of the differences caused by the different field distributions at different frequencies was an order of magnitude smaller than the results shown in Figs. 7–10. Thus, field differences caused by the different RF distributions at different frequencies can be neglected.

Figures 7 and 8 show the x component electric field difference maps in the xz and yz planes for the third and port accelerating cavities, respectively, and Fig. 9 shows the x component magnetic field difference maps for the port cavity.
along the same planes. The $xz$ plane shows the field differences in a plane including the coupling irises, while the $yz$ plane shows field differences in a plane not including coupling irises, but including the port iris for the port accelerating cavity. Only the $x$ component electric field differences are shown for the third accelerating cavity (AC3) since all other components, including all magnetic field components, show negligible differences in the beam tube (less than $10^3$ V/m). For the port cavity (AC4), the $x$ component magnetic field difference map is shown in addition to the $x$ component electric field map due to its large effect in the beam tube. As with AC3, the other field components are not shown since they yield negligible differences in the beam tube. In Figs. 8 and 9, some isolated numerical noise can be seen at the sharp edges caused by the intersection of the side cavity or port waveguide with the curved surface of the accelerating cavity. The sharp edges in the numerical model allow for a reduced number of mesh elements, significantly decreasing the memory and computational time requirements. In a manufactured linac, the edges would be smooth lowering the field concentration and reducing the probability of electrical breakdown.

In Figs. 7–9, the color scale was set such that field differences within the beam tube would be prominent while still showing changes throughout the entire cavity. The maximum magnitude of the electric field difference in the beam tube was of the order of $10^5$ V/m for both the third and port accelerating cavities while the maximum difference in the magnetic field was of the order of $10^3$ A/m. In order to get a feel for the magnitude of the differences, the electric field strength in a typical accelerating cavity (Fig. 6) is of the order of $5 \times 10^7$ MV/m and the magnetic field strength is $1 \times 10^4$ A/m. To further illustrate the magnitude of these differences, Fig. 10 shows the absolute difference for the $x$ component of the electric field for the length of the third cavity along the waveguide center. The noncoupled cavity has zero field magnitude of the $x$ component electric field due to cylindrical symmetry, but side coupling introduces a maximum magnitude of just under $-0.12$ MV/m on the axis.

In Figs. 7 and 10 it is clear that the $x$ component of the electric field is more negative throughout the cavity as a result of the coupling. As the electron beam traverses the cavity, it experiences an extra negative transverse force in the $x$ direction that is not present for non-side-coupled structures. This leads to a shifting of the electron beam in the $x$ direction, but the shift is not global since the differences introduced are asymmetric. For example, as seen in Fig. 7, the change in the $x$ component of the electric field due to coupling is different in the $xz$ plane compared to the $yz$ plane. In the $xz$ plane, the change caused by the coupling irises leads to an increasingly larger difference going away from the axis of the waveguide. The opposite is true in the $yz$ plane where

![Figure 6](image.png)

**Fig. 6.** The electric field solution within the simulated in-line side-coupled 6 MV linac waveguide incorporating 1.5 and 0.5 mm side cavity shift is given. The axial electric fields were taken on the central axis, while the radial electric fields were taken at the beam tube edge.

### Table II. A summary of all the waveguide dimensions that were optimized in the 1.5 mm (0.5 mm in brackets) cavity shift in-line side-coupled 6 MV linac model are given. The Greek letters refer to the dimensions outlined in Fig. 1. Dimensions that were not changed are designated by an en dash. These dimensions together with the dimensions in Fig. 1 specify the entire waveguide.

<table>
<thead>
<tr>
<th></th>
<th>Optimized AC dimensions (mm)</th>
<th>First AC dimensions (mm)</th>
<th>Port AC dimensions (mm)</th>
<th>End AC dimensions (mm)</th>
</tr>
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<tr>
<td>Inner corner radius ($\beta$)</td>
<td>5.50</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Cavity radius ($\gamma$)</td>
<td>38.46</td>
<td>38.325 (38.264)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Inner nose cone radius ($\delta$)</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Nose cone length ($\varepsilon$)</td>
<td>10.88</td>
<td>10.85 (–)</td>
<td>–</td>
<td>11.03 (11.03)</td>
</tr>
<tr>
<td>Beam tube diameter ($\zeta$)</td>
<td>5.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Post length ($\alpha$)</td>
<td>9.58</td>
<td>9.55 (–)</td>
<td>– (9.57)</td>
<td>–</td>
</tr>
</tbody>
</table>
the largest difference is on axis and becomes lesser near the beam tube edges. With a fully symmetric cavity, the electrons at \( \pm x \) and \( \pm y \) positions in the beam tube experience identical RF field magnitudes leading to a fully symmetric beam. The asymmetric differences seen in Figs. 7–9 mean that electrons at \( \pm x \) and \( \pm y \) positions in the beam tube no longer experience the same RF field, leading to a shift and skewing of the electron beam.

The effect of the asymmetries seen in Figs. 7 and 8 are minimized by the staggered location of the side cavities as seen in Fig. 2. Two accelerating cavities are coupled with the side cavity on top of the waveguide and the next two accelerating cavities are coupled with the side cavity at the bottom of the waveguide. The x component of the electric field seen in Fig. 7 is more negative due side coupling, whereas it is less negative for the adjacent port cavity in Fig. 8. Overall, the staggering of the side cavities along the top and bottom of the waveguide causes the polarity of these field asymmetries to be reversed in adjacent cavities for the same RF phase. However, a complete cancellation of the asymmetries is not expected since the effects on the low velocity electrons at the beginning of the waveguide will be greater than at the end where they are traveling near the speed of light.

The electric field differences in the port cavity show large changes in the cavity near the input port, but the effect is very small within the beam tube with the exception of the x component, and hence the electrons experience a nearly identical electric field in the port cavity compared to the nonport cavities. The effect of the port iris on the magnetic field however is non-negligible in the beam tube for the port cavity as seen in Fig. 9. The x component of the magnetic
field shows the largest changes due to the magnetic field's circumferential nature. The \( y \) and \( z \) components of the magnetic field are zeros or at least very nearly so at the port, while the \( x \) component is near maximum. Thus, the perturbations to the field will be most strongly experienced by the \( x \) component of the magnetic field. As seen in Fig. 9, the changes in the field are asymmetric within the beam tube again causing an asymmetric force to be imparted to the electron beam predominantly in the \( y \) direction according to the Lorentz force.

The particle simulations performed in PARMELA show the expected shifting and skewing of the electron beam due to the asymmetric RF field within the side-coupled linac waveguide. For all particle simulations, a beam loading power of 1 MW was calculated with a target current of 180 mA. Figure 11 shows the spatial electron intensity distribution at the target in the \( x \) and \( y \) directions for the 0.5 and 1.5 mm side cavity shift simulations with coupling along with a simulation performed without the effects of coupling. The peak positions as well as the center of gravity of the \( x \) and \( y \) distributions are all zero for the fully symmetric, noncoupled waveguide simulation. When the 0.5 mm side cavity shift waveguide with coupling was modeled, the distribution peak and center of gravity positions were calculated to be 0.02 and 0.008 cm, respectively, in the \( x \) direction, and −0.004 and 0.005 cm, respectively, in the \( y \) direction. For the 1.5 mm side cavity shift waveguide, the peak and center of gravity positions changed to 0.01 cm for both in the \( x \) direction and −0.005 and −0.007 cm, respectively, in the \( y \) direction. From this analysis, at an input power of clinical relevance to a medical in-line side-coupled waveguide, the greatest effect on a cylindrical beam is due to the side coupling. An increasing input power would cause an increasingly larger beam shift in the \( y \) direction, but for an input power from the magnetron powering the 600C linac, the presented shift is the expected clinical output for an injected cylindrical electron beam. From Fig. 11, it is also apparent that the larger RF field magnitudes in the first accelerating cavity cause a larger beam spot at the target. This may, in part, be due to a greater redistribution of the electron beam upon injection into the linac in order to shield the interior of the beam from the larger RF focusing forces.

The effect on the dose distribution from the small peak and center of gravity shifts of the electron beam due to side coupling is shown in Fig. 12. However, the dose distribution in the direction of the port iris (i.e., in the \( y \) direction) showed very little effect due to the port iris and is thus not shown here. Very little difference in the dose distributions for the waveguide designs between the 0.5 or 1.5 mm side cavity shift is seen, but the effect of side coupling as a whole exhibits itself as a 1% asymmetry in the profiles. This asymmetry is clinically acceptable, but, in practice, it is not seen after the linac has been properly commissioned. In the commissioning process, the waveguide is translated laterally with respect to the flattening filter until symmetric dose distributions are achieved, correcting the small effect of side coupling in the process. Despite the peak shift being larger in the 0.5 mm cavity shift design, the total beam spot is larger with a greater number of electrons left of the main peak in Fig. 11 keeping the beam center of gravity similar.
to the 1.5 mm side cavity shift model. The similar beam center of gravities most likely causes the dose profiles to be similar. As seen in Figs. 11 and 12, the effect of the side cavities and coupling port on a short in-line side-coupled linac has a very small effect on the dose distributions, which is easily removed during the commissioning process. For the linac modeled, and the ones similar in design and length, the effect of the coupling can be, in practice, ignored. However, longer waveguide structures like those for high energy medical beams cannot ignore these effects and, in practice, use solenoid focusing and steering coils to ensure the beam is not deflected to a large extent.

The exact side cavity shift and proper electron injector design for this waveguide to more exactly emulate a Varian 600C require the incorporation of various electrical measurements from a Varian 600C into the model and the Monte Carlo commissioning process to ensure that the model waveguide and the Varian 600C are dosemetrically equivalent. The focal spot size can be adjusted by changing the first side cavity shift as explained above and the beam energy can be adjusted by changing the RF magnitude within the waveguide according to the constraint that the required power must not exceed what is available from clinically used magnetrons. The full commissioning of the 3D in-line side-coupled waveguide is the subject of current work. With a fully commissioned 3D linac model, a better estimate of the extent of beam shifts due to the side-coupling and port irises of a Varian 600C will be known, as well as their effect on the dose distribution.

With a full 3D waveguide simulation, the electron trajectories can be accurately determined with and without the presence of an external magnetic field. In the pursuit of determining the maximum magnetic field in which the linac can operate, an analysis of asymmetries in dose profiles along with the effect of beam loss within the waveguide caused by an external magnetic field is required. If the shifting and skewing of the electron beam simply caused by the side and port coupling irises are not first taken into account, an underestimation of the effect of the external magnetic field is inevitable. The shift away from the central axis caused by an external magnetic field is increased due to the inherent effects of the side and port coupling irises. Thus, the designed 3D simulation outlined here is able to quantify the true effect on the electron beam in a side-coupled medical linac waveguide in the presence of an external magnetic field. It can thus be used to optimize magnetic shielding needed for the linac-MR system so that true real-time IGRT can be fully realized.

IV. CONCLUSIONS

A concise design for the generation of an accurate simulation of an in-line side-coupled 6 MV medical linac waveguide has been given. The design and optimization of the side-coupled waveguide given were designed to emulate a Varian 600C clinical waveguide. The 3D RF field solution within the reference cavity was shown to be highly accurate compared to the benchmarked program SUPERFISH, and the completed full waveguide simulation was able to incorporate the effects of side and port coupling. The effect of the side and port irises on the RF field have been quantified and have been shown to be predominantly in the electric and magnetic fields, respectively. As expected for the π/2 operating mode, the total energy contained within the coupling cavities was three orders of magnitude smaller than that in the accelerating cavities since these cavities contain the field nodes. Using the particle tracking program PARMELA, the electron trajectories within the simulated waveguide have been determined and have shown a slight shifting and skewing of the electron beam due to the effects of the side and port coupling irises. The dose profile in the direction of the coupling cavities was found to show the greatest change introducing a 1% asymmetry due to the side-coupling irises. However, in practice, this asymmetry is removed in the proper commissioning of the medical linear accelerator.

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