Design and Optimization of Superconducting MRI Magnet Systems With Magnetic Materials

Tony Tadic and B. Gino Fallone

Abstract—We present a method for the optimal design of superconducting magnet systems for magnetic resonance imaging (MRI). The method integrates a linear-programming technique with the finite-element method (FEM) to calculate minimum-volume coil configurations subject to magnetic field homogeneity constraints for MRI systems that contain general nonaxisymmetric magnetic yoke structures. The method rapidly converges and only requires a small number of iterations and FEM analyses to be performed. In particular, the method is well suited for magnet design problems that necessitate large 3-D FEM models. We demonstrate the method with the optimal design of an open and compact 0.5 T yoked biplanar magnet assembly considered for use in an integrated medical linear accelerator and MRI system. In particular, the coil configuration for this magnet design is constructed from a MgB$_2$ high-temperature superconducting material that operates in a conduction-cooled cryogen-free environment.

Index Terms—Image-guided radiation therapy, magnet design, magnetic resonance imaging (MRI), optimization, superconducting magnets.

I. INTRODUCTION

SUPERCONDUCTING magnet systems are currently the most widely used magnet types for generating the strong homogeneous static magnetic field required for clinical magnetic resonance imaging (MRI). These systems are capable of greater field strengths, better temporal stability, and higher field uniformity than their permanent and resistive magnet counterparts. These highly advantageous features permit faster imaging, greater spatial resolution, and superior image quality.

The optimal design of superconducting MRI magnets has been the subject of considerable research. Many approaches that focus on the optimization of coil configurations to yield compact shielded magnets that exhibit a high degree of uniformity and confined fringe fields have been reported [1]–[11]. Despite yielding significantly different coil arrangements, the majority of the magnet designs resulting from these methods achieve similar performance [12]. Consequently, there has been a recent thrust to further develop techniques that also minimize the conductor volume of the coil configuration [13]–[25]. Because the cost of a superconducting magnet system is strongly related to the amount of the conductor used, this condition has become a growing factor of importance in the increasingly competitive market for MRI magnets.

The majority of optimization algorithms for superconducting magnets only consider designs that either have a cylindrical geometry [1], [25]–[34] or do not contain any magnetic materials [2]–[11], [13]–[24], thereby permitting a simplification of the required magnetic field analysis. This case was largely justified by the fact that most commercial full-body magnets satisfied these assumptions [12]. Field calculations for coil-only systems can rapidly be performed with formulas that are derived from the Biot–Savart law or by employing well-known spherical harmonic expansions [35]. When considering magnetized materials with nonlinear susceptibilities, the magnetic-field analysis becomes much more complicated. Equivalent magnetization current methods and other direct techniques utilizing analytical formulas for the magnetic fields due to magnetized rings have been proposed [29]–[32], although these schemes are strictly limited to cylindrical geometries. The finite-element method (FEM) has also been applied in problems that involve magnetic materials due to its high accuracy and ability to handle complicated geometries. However, to reduce the number of mesh elements and the associated computation time, the majority of optimization methods that employ the FEM assume simplified cylindrical geometries, permitting the use of smaller 2-D models [26]–[28], [33], [34].

Corresponding to the current trend toward improving patient access and decreasing patient claustrophobia, there is a growing demand for compact open biplanar magnet systems [12], [36], [37]. These magnets typically incorporate noncylindrical yoke structures, for which accurate analysis of the 3-D magnetic fields and associated inhomogeneities are required during the design process. We are therefore motivated to develop new optimization methods that can be applied to the design of these systems.

In this paper, we present an iterative method for the optimal design of homogenous superconducting magnet assemblies with general nonaxisymmetric magnetic yoke structures. A linear-programming (LP) subproblem is solved at each iteration to obtain a minimum-volume coil configuration while constraining the magnetic-field inhomogeneity within an arbitrary target volume [14]. The FEM is then used to calculate the complete 3-D magnetic field produced by the entire magnet...
The design process begins with user specification of the magnetic yoke structure, for which the geometry and properties of the materials involved are held constant for all subsequent steps. The user then defines the feasible coil domain and a distribution of \( N_j \) target points at locations where the magnetic field uniformity will be constrained. These points are typically chosen on the surface of a large diameter spherical volume (DSV) that is designated for imaging; thus, the constraints are expressed as

\[
|b_j - B_0| \leq \varepsilon B_0 \quad j = 1, \ldots, N_j
\]

where \( b_j \) is the total axial component of the magnetic field at the target point \( j \), \( B_0 \) is the desired magnetic field strength at the isocenter, and \( \varepsilon \) is the relative field error tolerance. The radial component of the magnetic field can be considered negligible when the axial component is homogenous [14]; therefore, it is not included in the constraint formulation.

It is assumed with this design method that the yoke structure itself provides adequate reduction of the magnetic fringe fields produced by the magnet assembly, as typically found with modern passively shielded biplanar MRI systems [12]. Consequently, we do not employ shielding constraints when optimizing the coil configuration.

For convenience, \( b_j \) can be separated into two components as

\[
b_j = b_{cj} + b_{mj} \quad j = 1, \ldots, N_j
\]

where the coil field \( b_{cj} \) is the contribution strictly due to the free-current configuration, and the material field \( b_{mj} \) is the contribution due to the nonlinear magnetization induced in the magnetic materials. The target coil field \( b_{tj} \) can then be defined as

\[
b_{tj} = B_0 - b_{mj} \quad j = 1, \ldots, N_j
\]

and the constraints (1) may be rewritten as

\[
|b_{cj} - b_{tj}| \leq \varepsilon B_0 \quad j = 1, \ldots, N_j.
\]

A coil configuration is not initially defined, and thus, the magnetic field is zero at every target point, i.e.,

\[
b_{cj}^{(0)} = b_{mj}^{(0)} = 0 \quad j = 1, \ldots, N_j
\]

where the superscript denotes the iteration number. Because the constraints (4) are not initially satisfied, the iterative design loop is engaged.

During the first step of each iteration, an LP method adapted from [14] is used to determine a minimum-volume coil arrangement such that (4) is satisfied. The details of this procedure are discussed in the following section. Because the true material field is unknown at this stage, the target coil field in (4) is calculated using the material field from the previous iteration as an estimate [27], [28]. Therefore, the coil configuration obtained with the LP method at the \( n \)th iteration satisfies

\[
|b_{cj}^{(n)} - b_{tj}^{(n-1)}| \leq \varepsilon B_0 \quad j = 1, \ldots, N_j
\]

where the target coil field is given by

\[
b_{tj}^{(n-1)} = B_0 - b_{mj}^{(n-1)} \quad j = 1, \ldots, N_j.
\]
where iteration is now easily determined as target points, and coil field is already known, the true material field at the axial field at each of the target points is calculated. Because the specified yoke design is then generated, and the complete A FEM model that combines the new coil configuration with here as centered on the $z$-axis. The target points at which the field homogeneity is constrained are typically distributed on the surface of a large DSV designated for imaging.

A FEM model that combines the new coil configuration with the specified yoke design is then generated, and the complete axial field at each of the target points is calculated. Because the coil field is already known, the true material field at the $n$th iteration is now easily determined as

$$b_{m_j}^{(n)} = b_j^{(n)} - b_{t_j}^{(n)} \quad j = 1, \ldots, N_j. \quad (8)$$

The true material field typically differs from the estimate used in (6) and (7); therefore, the constraints (4) are no longer satisfied. However, the difference in the material fields between successive iterations tends to progressively decrease. In practice, the stopping criteria is slightly relaxed so that an excessive number of iterations is avoided, and therefore, the process is repeated until

$$|b_{m_j}^{(n)} - b_j^{(n)}| \leq \gamma \epsilon B_0 \quad j = 1, \ldots, N_j \quad (9)$$

where the convergence factor $\gamma$ is typically chosen between 1 and 3.

**B. LP Formulation**

At each iteration of the proposed method, we encounter the problem of determining a minimum-volume coil configuration that satisfies the field uniformity constraints (6). To accomplish this task, we begin by segmenting the specified feasible coil domain with a dense grid that defines an array of $N_k$ candidate coils, each of which is located at the center of a grid cell (see Fig. 2). These grid cells do not represent physical cross sections of the candidate coils but, rather, only serve to define their spacing and location.

We can now define the axial magnetic-field matrix $A$, for which the element $A_{jk}$ is the field per unit current at the $j$th target point due only to the $k$th candidate coil. This condition allows us to write the matrix expression:

$$b_c = Ai \quad (10)$$

where $b_c$ is the vector of axial coil magnetic fields at the $N_j$ target points, and $i$ is the vector of currents in the $N_k$ coils.

The total conductor volume $V$ in the array of $N_k$ circular coils is given by

$$V = \sum_{k=1}^{N_k} 2\pi r_k S_k \quad (11)$$

where $r_k$ and $S_k$ are the radius and cross section of the $k$th coil, respectively. Assuming a constant current density of $J$ in all of the candidate coils, the individual currents satisfy

$$|i_k| = JS_k \quad (12)$$

where $i_k$ is the current in the $k$th coil. Hence, the total volume can be rewritten as

$$V = 2\pi J \sum_{k=1}^{N_k} r_k |i_k| \quad (13)$$

As pointed out in [14], this expression is valid for any series-connected coil configuration employing a uniform wire size. Combining (6) and (10), the coil volume minimization problem at the $n$th iteration may be expressed as

Minimize : $\Psi = \sum_{k=1}^{N_k} r_k |i_k|$ \quad (14a)

Subject to : $b_t^{(n-1)} - \epsilon B_0 \leq Ai \leq b_t^{(n-1)} + \epsilon B_0$. \quad (14b)

This expression is an $l_1$-norm minimization problem, which can be rewritten in canonical LP form and solved for the globally optimal currents with the method presented in [14]. One beneficial feature of this problem is that it yields a sparse solution, for which the minimum number of nonzero currents required to satisfy the homogeneity constraints are obtained. In solving the minimization problem (14), we use the built-in function linprog from MATLAB (version 7.11, The MathWorks Inc., Natick, MA), which employs an interior-point predictor–corrector algorithm based on the linear-programming interior-point solvers (LIPSOL) method [41].

When performing the magnetic field calculations required when assembling $A$, we initially approximate each of the candidate coils by an ideal current loop with a zero cross-sectional area. This approach permits the use of simple axial field formulas involving complete elliptic integrals of the first and second kind [42]. When iterating on finer coil grids based on the procedure described in [14], the method in [43] is used to calculate the field due to real coils with finite rectangular cross-sections. The required cross-sectional areas (and dimensions) of the real coils are determined from the optimal currents using an aspect ratio specified by the magnet designer prior to optimization.

The yoke geometry is not included in the aforementioned optimization process. Therefore, the resulting designs obtained with this method are suboptimal. Because the weight and cost of these systems are strongly related to the design of the yoke structure, incorporating the nonlinear optimization of the magnetic materials within this scheme is a subject of future interest.
III. OPTIMIZATION EXAMPLE

The method described in this paper has been applied to the optimal design of an open compact 0.5 T biplanar superconducting magnet assembly for potential use in the linac–MRI system currently pursued by our group [38]–[40]. The geometry of the magnet assembly and linac arrangement is illustrated in Fig. 3.

This system is designed to employ a conduction-cooled cryogen-free coil configuration constructed from a MgB$_2$ HTS material. In the absence of any liquid coolant, we eliminate the requirement of a bulky cryostat vessel that consists of a safety ventilation system, thereby making the mechanical rotation of the entire scanner feasible.

An operating coil temperature of approximately 12 K could be achieved using two double-stage Gifford–McMahon (GM) cryocooler subassemblies in a conduction-cooled split cryostat design similar to the designs detailed in [44] and [45]. Within a vacuum chamber that encompasses each of the magnet poles, a thermal screen that surrounds the superconducting coil arrangement would be thermally coupled to the first stage of a single GM cryocooler. The second stage of this cryocooler could then be coupled to a set of thermally conductive rings and sheets embedded within and bonded adjacent to the coil forms and windings. It is expected that such a cryostat design and associated support structures would occupy an additional 3–4 cm of space surrounding the coils. By employing superconducting joints recently developed for MgB$_2$ tapes [46], this design permits a persistent mode of operation.

A realistic target for the engineering critical current density for MgB$_2$ is 28 kA/cm$^2$ at 12 K and 4 T [24]. Based on this value, we specify a working current density of 21 kA/cm$^2$ at approximately 75% of the critical value.

The outer dimensions of the four-column yoke structure illustrated in Fig. 3 have been chosen to permit the rotation of the magnet assembly within a typical radiotherapy vault of a 3 m height. In particular, a practical clearance of 40 cm is achieved for the magnet to allow room for additional peripheral equipment [36], [37]. The remaining dimensions were selected such that the total weight of magnetic material was within 10% of the 20 t yoke structure used in a commercially available conduction-cooled 0.5 T biplanar system with an acceptable fringe magnetic field [47].

The yoke and column structures of the magnet assembly are composed of AISI 1020 plain carbon steel, and the pole plates are composed of special Armco magnetic steel. The nonlinear magnetization curves for these materials are shown in Fig. 4 [48], [49].

The feasible coil domain is defined in the region surrounding the pole plates such that adequate space is supplied for the cooling and mounting system while providing a patient gap of approximately 60 cm (see Fig. 3). An initial array of 192 circular candidate coils with a grid spacing of 2 cm is employed. This grid spacing is further reduced to 1–2 mm during the refinement process in the LP coil optimization method [14]. Because the coil configuration in this particular magnet design is completely axisymmetric, only axisymmetric field inhomogeneities may be constrained. Accordingly, a distribution of 34 target points is defined along a single polar arc located on the surface of a 40 cm DSV at the isocenter, as shown in Fig. 5.

Once the minimum-volume coil configuration at each iteration is obtained, a FEM model is generated and analyzed to determine the complete magnetic fields produced by the
entire magnet assembly. In our current implementation, we use the 3-D FEM software package Opera-3d (version 13.0, Cobham plc, Wimborne Minster, England), which employs the magnetostatics solver TOSCA to calculate the scalar magnetic potential and magnetic field values at discrete points within the model geometry [50]. The updated parameters that define the coil configuration are passed to Opera-3d using a component object model (COM) interface that is established within the MATLAB script. This way, updating the complete FEM model at each iteration is an automated step in the optimization. Due to symmetry, only 1/8 of the magnet structure is included in the model geometry, which is partitioned with $1.017 \times 10^6$ tetrahedral quadratic Lagrange elements. The simplified model geometry and FEM mesh is shown in Fig. 6.

When executing the optimization method, the target field $B_0$ is set to 0.5 T, with a relative field error $\varepsilon$ of $10^{-5}$ and a convergence factor $\gamma$ of 2, resulting in an expected peak-to-peak field error of at most 40 ppm at the target points in the final design.

The optimal magnet design illustrated in Fig. 7 is obtained in 11 iterations. The dimensions of the resulting coil configuration are detailed in Table I. The total conductor volume in the 12 circular coils is $1.791 \times 10^4$ cm$^3$. A central magnetic field

**TABLE I**

<table>
<thead>
<tr>
<th>Coil</th>
<th>Radial position (mm)</th>
<th>Axial position (mm)</th>
<th>Radial width (mm)</th>
<th>Axial width (mm)</th>
<th>Current (kA-turns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>181.1</td>
<td>381.1</td>
<td>3.7</td>
<td>3.1</td>
<td>-2.416</td>
</tr>
<tr>
<td>2</td>
<td>194.4</td>
<td>341.1</td>
<td>6.7</td>
<td>5.6</td>
<td>7.909</td>
</tr>
<tr>
<td>3</td>
<td>245.6</td>
<td>341.1</td>
<td>10.8</td>
<td>9.0</td>
<td>-20.416</td>
</tr>
<tr>
<td>4</td>
<td>324.9</td>
<td>341.1</td>
<td>16.4</td>
<td>13.6</td>
<td>46.888</td>
</tr>
<tr>
<td>5</td>
<td>461.1</td>
<td>341.1</td>
<td>29.5</td>
<td>24.6</td>
<td>-152.608</td>
</tr>
<tr>
<td>6</td>
<td>658.9</td>
<td>341.1</td>
<td>42.7</td>
<td>35.6</td>
<td>319.034</td>
</tr>
</tbody>
</table>

Due to symmetry, only the dimensions for the upper pole are shown.

**Fig. 7.** Complete optimal coil configuration. The axial magnetic field variation (in parts per million) is plotted over the surface of a 40 cm DSV at the isocenter. Only a quarter section of the yoke geometry is shown for clarity. Axis dimensions are given in meters.

**Fig. 8.** Total peak-to-peak axial magnetic field inhomogeneity measured in parts per million as a function of the DSV size.
Fig. 9. Fringe field extent of the optimized magnet. The 5 G magnetic-field line is plotted in the $xz$ plane. Axis dimensions are given in meters.

### TABLE II

**SPHERICAL HARMONIC DECOMPOSITION OF THE AXIAL MAGNETIC-FIELD INHOMOGENEITY**

<table>
<thead>
<tr>
<th>Order</th>
<th>Degree</th>
<th>Harmonic amplitude (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>115</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-52</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>98</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>-21</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>-45</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

Amplitudes calculated over a 40 cm DSV

...strength of 0.500 T is achieved, and the peak-to-peak axial field inhomogeneity is 26.1 ppm at the target points, therefore satisfying the specified design constraints. The total inhomogeneity over the entire 40 cm DSV is 235.0 ppm, which is inherently limited by the nonaxisymmetric nature of the yoke structure. A result of this magnitude is typical for the theoretical design of biplanar magnets with nonaxisymmetric yoke geometries [51], [52], for which achieving a uniform magnetic field is generally more difficult than with a cylindrical magnet. The use of postmanufacturing shimming techniques would be necessary to further reduce the remaining inhomogeneities to a level suitable for medical imaging.

The total inhomogeneity as a function of the DSV size is shown in Fig. 8, and the extent of the fringe field is shown in Fig. 9. The peak field at the coils is 3.428 T, which satisfies the specifications for the conductor material chosen. For completeness, the harmonic amplitudes for the spherical harmonic decomposition of the axial magnetic field inhomogeneity over a 40 cm DSV at the isocenter are provided in Table II.

It has been demonstrated that the linac waveguide is sensitive to the transverse magnetic fields that it experiences at the location illustrated in Fig. 3 [53]. Accordingly, additional linear constraints similar to the constraints in (1) could be defined such that the strength of the magnetic field along the linac would be limited during the aforementioned optimization process. Shielding the linac with this approach would require an increase in the coil currents and would result in higher peak fields. Hence, simpler alternative approaches involving a dedicated shielding system are preferred [54].

### IV. CONCLUSION

An iterative method has been presented for the optimal design of homogenous superconducting MRI systems that contain magnetic materials. In addition to minimizing the conductor volume, one key feature of this method is its ability to handle magnet designs involving complicated noncylindrical magnetic yoke geometries, such as those found in modern passively shielded biplanar MRI scanners. The accurate design of these systems requires the use of computationally expensive numerical techniques for the magnetic field analysis, the burden of which is alleviated by the rapid convergence achieved with the proposed method.

### REFERENCES
